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Finite Element Analysis

Date Jan 19, 2023

Strong Form - Chapter 3

Start w/ Strong form → Weak Form

Chapter 3 — Chapter 3

ODE / PDE

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Approximations — Finite

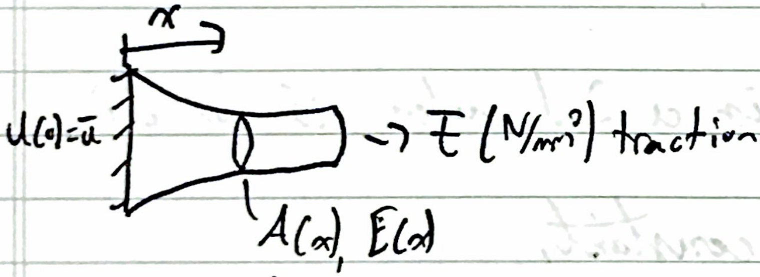
Chapter 4 — Chapter 5+

SF = governing DE

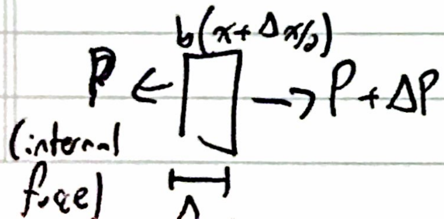
↳ has boundary conditions and initial conditions (time)

↳

Eg. Elastic Body in 1D



- Consider equilibrium of small slice Δx



, $P = \sigma A$

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Note - equilibrium req's

- $\sum F_x = 0$, $\therefore (P+\Delta P) + b\Delta x - P = 0$

$$\frac{\Delta P}{\Delta x} + b = 0$$

$\lim_{\Delta x \to 0}$, we obtain $\frac{dP}{dx} + b = 0$

Noting that Hooke's Law $= \sigma = E\epsilon$

- ϵ = elongation per unit length, $\frac{\text{elongation}}{\text{initial length}} = \lim_{x \to 0} \frac{[u(x+\Delta x) - u(x)]}{\Delta x}$

$$= \frac{du}{dx} = \epsilon$$

u = displacement at x

Combining,

$$\frac{d}{dx}\left(AE\frac{du}{dx}\right) + b = 0.$$

↳ This is a 2nd order ODE for $u(x)$

If AE is constant,

$$AE\frac{d^2u}{dx^2} + b = 0, \text{ shortened}$$

↳ $AEu_{,xx} + b = 0$

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$$\frac{du}{dx} = -\int_0^x \frac{b(x)}{EA}\,dx + C_0$$

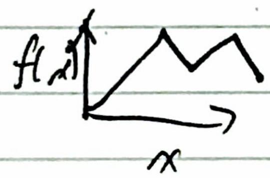
$$u(x) = -\int_0^x \int_0^x \frac{b(x)}{EA}\,dx\,dx + C_0 x + C_1$$

Continuity of Functions

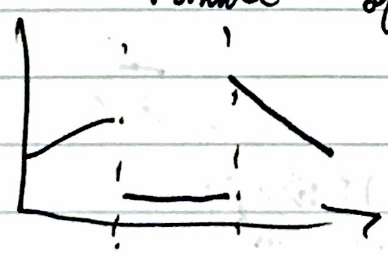
C^2 - function Space
↳ Set of functions w/ continuous 2nd derivatives

C^1 - Set of " " " 1st "

C^0: " " " that are continuous



C^{-1} - Set of all piecewise continuous functions
↳ Finite # of points



From before, $AE u_{,xx} + b$ is a C^2 function

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Elastic Strong Form

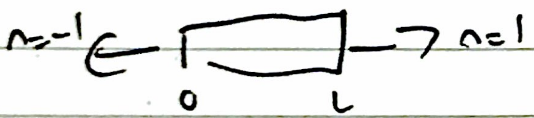
1. Find $u(x) \in C^1$ such that

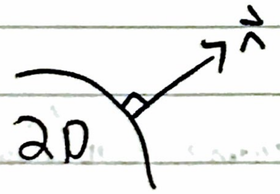
$$\frac{d}{dx}\left(AE\frac{du}{dx}\right) + b = 0 \;, \quad 0 \le x \le L$$

$u(x=0) = \bar{u}$

$\bar{t} = \left. \sigma \cdot n \right|_{x=L}$ when $n=-1$ @ $x=0$
$n=1$ @ $x=L$

n is the normal to the boundary of the domain

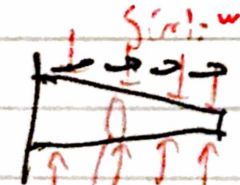




Heat Transfer

$s(x)$: watts/meter

$T = \bar{T}$



$\rightarrow \bar{q}$ (heat flux) $\left[W/m^2 \right] \rightarrow J/s$

$A(x)$, K = thermal conductivity $\left[\frac{W}{m^2\text{-}K} \right]$

Kelvin

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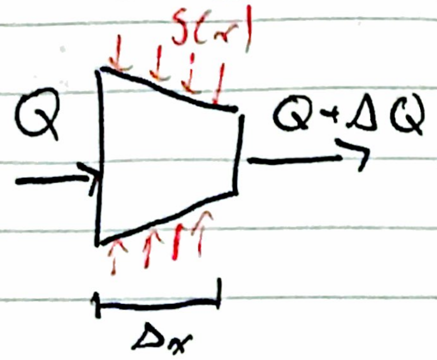
Consider a slice



, $Q = Aq$

Using equilibrium (conservation of E), assuming steady state,

$$Q - (Q + \Delta Q) + S\Delta x = 0$$

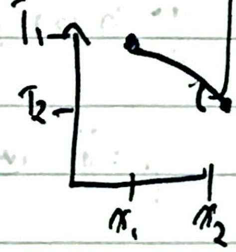
$$-\frac{\Delta Q}{\Delta x} + S = 0$$

in the limit as $\Delta x \to 0$,

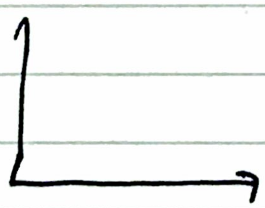
$$-\frac{d}{dx}\left(Aq\right) + S = 0$$

Fourier's Law

$q = -k \dfrac{dT}{dx}$ → Goes from hot to cold.



Slope negative ∴ $\frac{dT}{dx}$ is negative



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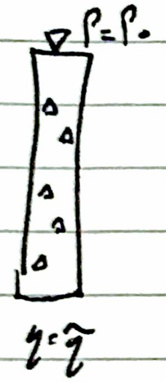
Strong Form

Find $T(x) \in$ such that $\frac{d}{dx}\left(Ak\frac{dT}{dx}\right) + s = 0, \quad 0 \le x \le L$

$T(0) = \bar{T}$

$q \cdot n \big|_{x=L} = \bar{q}$

In Porous Media



Find $p(z) \in C^1$ such that:

$\frac{d}{dx}(-Aq) + s = 0,$

q: mass flow: Darcy's Law $= \frac{-k}{\mu}\frac{dP}{dz}$

$p = \bar{p}$ @ $z = 0$

$q = \frac{-k}{\mu}\nabla p$

$q \cdot n = \tilde{q}$ @ $z = L$

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