

$$\frac{d}{dx} \left( AE \frac{du}{dx} \right) + 10Ax = 0, \quad 0 < x < 2, \quad u(0) = 10^{-4}, \quad \bar{F} = 10 = E \frac{du}{dx} \Big|_{x=2}$$

$$1. \quad \int_0^2 \left[ \frac{d}{dx} \left( AE \frac{du}{dx} \right) + 10Ax \right] dx = 0, \quad \forall u(x)$$

$$\int_0^2 \left[ w E \frac{du}{dx} \Big|_{x=2} - 10Ax - 10 \right] dx$$

$$= wA \left[ E \frac{du}{dx} \right]_{x=2} - wA10 = 0 \quad \forall w(x)$$

$$2. \quad \text{IBP}, \quad u dv = uv - v du$$

$$\int_0^2 w(x) \frac{d}{dx} \left( AE \frac{du}{dx} \right) dx + \int_0^2 10Ax dx, \quad \frac{d(w(x))}{dx} = \frac{dw}{dx}$$

$$d(w(x)) = \frac{dw}{dx} dx$$

$$\int_0^2 \frac{dw}{dx} AE \frac{du}{dx} dx$$

$$\frac{d(w(x))}{dx} = \frac{dw}{dx}$$

$$w(x) AE \frac{du}{dx} \Big|_0^2 - \int_0^2 AE \frac{du}{dx} \frac{dw}{dx} dx, \quad \text{since } w(0) = 0$$

$$d(w(x)) = \frac{dw}{dx} dx$$

$$w(2) AE \frac{du}{dx} - \int_0^2 AE \frac{du}{dx} \frac{dw}{dx} dx + \int_0^2 w(x) 10A dx, \quad \text{since } E \frac{du}{dx} = \bar{F}$$

$$w(2) \bar{F} A - \int_0^2 AE \frac{du}{dx} \frac{dw}{dx} dx + \int_0^2 w(x) 10A dx = 0$$

we're done